# Research Status and Progress of Portfolio Theory

**Lejia Zhang**

Guangdong Zhong Yuan Highschool, China

**Keywords:** Liquidity; Value at Risk; Behavioral Portfolio; Non-Utility.

**Abstract:** Portfolio theory has been born for half a century, but the research on it still emerges in endlessly. In recent years, with the requirements of investment practice and the development of mathematics and computing technology, portfolio theory has also appeared some new branches and development trends. This paper reviews the recent progress of portfolio theory in the past ten years, which can be divided into six categories: behavioral portfolio theory, non-utility portfolio theory, style portfolio theory, portfolio theory with liquidity, portfolio theory with continuous long-term and VaR portfolio theory. And elaborated their main contents, finally, briefly commented on these theories.

## 1. Introduction

Markowitz's paper *Portfolio Selection* published in 1952 and his monograph *Portfolio Selection: Effective Decentralization* published in 1959 marked the emergence of portfolio theory. Early studies on portfolio theory mostly focused on various single-period or multi-period portfolio problems under discrete-time conditions, but since MerTon [1,2] first examined portfolio problems under continuous-time conditions. With the rapid development of stochastic control theory, stochastic calculus and computer technology, the problem of a portfolio under continuous time has become a research hotspot. In recent years, the rise of Value-at-Risk (VaR) method and behavioral finance theory has also penetrated the field of portfolio theory, thus opening a new world for portfolio theory research.

Before the development of portfolio theory, the concept of diversified investment existed, but at that time investment management focused on the simple set of individual management. Later, the introduction of uncertainty played an important role in the development of portfolio theory. As early as the 1930s, Kenes (1936) and Hicks (1939) put forward the concept of risk compensation, believing that due to the existence of uncertainty, certain risk compensation should be added to different financial products in addition to interest rates. Hicks also raised the issue of asset selection, believing that risk can be dispersed. Then Von Neumann (1947) applied the concept of expected utility to propose a decision-making method under uncertain conditions. On this basis, American economist Markowitz published his paper *Portfolio Selection* in 1952, marking the beginning of portfolio theory. He uses the mean-variance model to conclude that portfolio investment can effectively reduce risk. At the same time, Roy (1952) proposed the Safety First Portflio Theory, which chooses the mean and variance of the portfolio as a whole. In particular, he proposed that the decision criterion of the model is to minimize the probability that the return of the portfolio is less than the given "disaster risk level". VaR (Value at Risk) and other methods provide ideas. Then Tobin (1958) put forward the famous "separation theorem of two funds": in the problem of portfolio selection which allows short selling, every effective portfolio is a combination of riskless assets and a special risky asset.

Sharpe, Lintner and Mossin put forward their respective capital asset pricing models (CAPM) in 1964, 1965 and 1966 respectively. These models are to explore the theory of asset pricing under uncertain conditions and have important guiding significance for investment practice. After the capital asset pricing model was put forward, researchers in the 1970s further deepened and enriched the research. In 1970, Brennan proposed a capital asset pricing model considering the effect of tax rate on the return of securities investment. Vasicek and Black studied capital asset pricing models in

 

1971 and 1972, respectively when there was no risk-free lending. In 1972, Mayers proposed the establishment of an asset pricing model considering the existence of non-market assets such as pensions and social insurance. Merton (1973) expanded CAPM to consider only the optimal asset selection in a single period and proposed a multi-factor ICAPM (Intertemporal CAPM) model, which laid the foundation for the later long-term investment theory.

The most revolutionary events in the 1970s are undoubtedly the option pricing formula derived by Blake and Scholes (1973), and the development and deepening of Merton (1973). Black-Scholes model, the option pricing formula, has weak application conditions. The underlying basic economic mechanism is no arbitrage principle. Because of its wide application, it can be used not only in option pricing but also in various forms of financial derivatives and the valuation of corporate debt. In the late 1970s, Harrison and Kreps (1979) developed the theory of martingale pricing, which is still a frontier topic in financial research.

## 2. Behavioral Portfolio Theory

Behavioral finance, which originated in the 1950s, has been playing an increasingly important role in financial theory. Behavioral finance holds that the behavior of irrational traders is not unpredictable, but restricted by psychological laws. If we can grasp their behavioral laws, we can find the corresponding profit-making methods. Behavioral finance put forward "Behavioral Portfolio Theory" (Shefrin and Statman, 1994), which is based on "Psychological Account". It holds that in reality, most investors form a pyramid-like behavioral portfolio because they have a different understanding of the risk degree of assets and different investment objectives. The assets at each level of the pyramid are linked to an objective and risk attitude, while the correlation between the layers is ignored by general financial theory. People will put their portfolio into two or more psychological accounts in their investment, which can effectively reduce the complexity of decision-making and the difficulty of execution, so it has more practical value. Shefrin and Statman [3] put forward the Behavioral Portfolio Theory (BPT) based on SP/A theory [4], Kahneman and Tversky expectation theory [5]. BPT has two forms: BPT-SA (a single mental account BPT) and BPT-MA (a multiple mental account BPT). BPT-SA is based on SP/A theory, while BPT-MA adds the psychological account structure of expectation theory.

Both BPT-SA and mean-variance investors consider the covariance and regard the portfolio as a whole, i.e. as a single psychological account. The cornerstone of mean-variance theory is the effective frontier of mean difference in ($\mu$, $\delta$) space, while BPT-SA is in ($E_h(W)$, $Pr\{W \leq A\}$) space, where A is the level of desire and $E_h$ (W) is the value of wealth expectation. In both cases, investors prefer higher $\mu$ and higher $E_h$ (W), lower $\sigma$ and lower$Pr\{W \leq A\}$. Obviously, for a fixed $Pr\{W \leq A\}$, the BPT-SA frontier is obtained by maximizing $E_h$ (W). However, the combination of BPT-SA effective frontier is generally different from that of mean-variance effective frontier. Shefrin and Statman have proved the following conclusion: if at least three states satisfy positive consumption and have different $v_i/p_i$ values in BPT-SA efficient combination, then the combination is not mean-variance efficient, and $v_i/p_i$ is the probabilistic price state per unit.

Most investors want to avoid poverty and get rich overnight. The combination of high and low aspirations is often described as a hierarchical pyramid, where investors distribute their wealth between the lower (to avoid poverty) and the higher (to become rich overnight). The fundamental difference between BPT-MA and BPT-SA is psychological account. BPT-MA investors seem to ignore covariance and act. They divide their portfolio into different psychological accounts. Tversky and Kahneman [6] point out the difficulties of covariance and other properties of joint probability distribution applied to psychological processes. By dividing joint probability distribution into psychological accounts, people simplify their choice in the combined hierarchical pyramid structure. A considerable number of experiments and practices have proved that investors ignore covariance [7,8]. Shefrin and Statman gave BPT-MA under two psychological accounts. Imagine that investors have three entities: a principal (planner) and two agents (practitioners). This follows the self-control framework proposed by Thaler and Shefrin [9]. The first practitioner has a low level of desire, the second has a high level of desire; each practitioner is associated with a psychological account. The planner distributes the current wealth W between the two practitioners. Balance the two to maximize total utility. Shefrin and Statman deduced the utility functions of planners under the assumption that the utility functions of the two practitioners were Cobb-Douglas functions, and analyzed the distribution equilibrium when short selling was allowed. At the same time, Engel curve is used to explain why low-desire accounts prefer risk-free bonds to high-desire accounts, while high-desire accounts prefer lottery tickets, while cautious optimists prefer the extreme combination of two levels, which shows that BPT-MA investors do not follow the separation theorem of two funds. At the same time, they draw the following conclusion: if at least five states of BPT-MA combination satisfy positive consumption and have different $v_i/p_i$ values, then the combination is neither mean-variance effective nor BPT-SA effective.

## 3. Non-utility Portfolio Theory

Early and previous portfolio theory is based on expected utility maximization. As an alternative to expected utility maximization, non-utility-based portfolio optimization methods have also been developed. Following are several main portfolio theory models based on non-utility.

Cover [10] develops a pan-combination model under discrete-time conditions. Pan-combination is optimal in the following sense: if the time range is large enough, pan-combination will behave (approximate) as a good constant rebalancing combination. A very attractive feature of pan-portfolio is that it does not need to know market parameters and relevant statistical information, such as interest rates, volatility, or even to describe the specific model of price dynamics in discrete time. Pan-portfolio is the equal wealth ratio of all securities, and then the weighted average of all the invariant portfolios (the performance of specific securities determines the weight). Its advantage is that it can track the optimal constant portfolio by adjusting the performance weights of different securities.

The cover also describes the asymptotic behavior of pan-portfolio and gives some numerical examples using market data. The results of the examples prove the good performance of pan-portfolio. Jamshidian [11] gives some conclusions about constant rebalancing combinations under continuous time conditions, and points out: it is good to calculate and follow the asymptotically optimal constant rebalancing portfolio $\pi^\infty$ but this requires full knowledge of the square of future instantaneous returns and long-term covariance, and to follow every other constant portfolio strategy will lead to exponential low performance (compared with using $\pi^\infty$). To overcome the danger of choosing "wrong" constant combinations, Jamshidian suggests using pan-combinatorial continuous-time variables. Finally, the literature draws the following conclusions: the performance of asymptotically optimal constant rebalancing Portfolio $\pi^\infty$ is only polynomially higher than that of pan-portfolio. Therefore, in the long run, pan-portfolio performance is also exponentially higher than all other constant portfolios, and pan-portfolio does not need advance information about market parameters to achieve this performance, which clearly shows the practical significance of pan-portfolio.

Buckley and Korn [12] investigated the exponential tracking problem with stochastic cash flow. For fund managers who try to track the index negatively, the portfolio would ideally consist of all the holdings of securities entering the index. Due to irregular cash inflows and outflows (generated by dividends, new fund subscriptions and fund redemptions), fund managers hold a certain amount of cash to save transaction costs. This naturally leads to tracking errors, i.e. the deviation between the performance of investors' Holdings (portfolios and cash accounts) and the performance of the index. Buckley and Korn give the relevant models in this case, solve the impulse control problem caused by fund managers, give the conditions for the existence of optimal control strategies, and prove the existence of optimal QVI (quasi-variational inequalities) control.

Hellwig [13] criticizes the portfolio theory of maximizing expected utility (consumption and end-of-life wealth). The general method of discounting future payments to add up consumption and end-of-life wealth time is unacceptable. The discount makes later time periods less important so that investors now consume too much. Therefore, the "economic power" of the portfolio decreases over time. Hellwing has developed a widely applicable pricing method for economic resources, Value Preserving Principle, that is, the intrinsic value of resources (future earnings value) changes over time. This method can be applied to portfolio optimization to make it more practical. Hellwing uses this method to study the portfolio optimization of financial markets in discrete time and finite state space. Korn gives a general continuous-time model based on semi-martingale, including Black and Scholes models. Korn [14] investigates the continuous-time model of price given by general diffusion process, explores the existence and uniqueness of market value maintenance strategies for some diffusion types, and gives examples of incomplete Black and Scholes models. The homogeneity of the portfolio process indicates that value maintenance can be interpreted as maximizing wealth growth rate in every instant. In this sense, it seems to be a short-sighted strategy. However, in the case of value maintenance, different consumers produce intertemporal rebalancing at different moments of time. Korn also gives the only value maintenance measure from the option hedging theory in incomplete markets, i.e. the minimum martingale measure, while the effect of additional constraints on portfolio strategy is examined in Korn.

## 4. Portfolio Investment Theory of Styles Investment

Style investment began with William Sharp's paper *Asset Allocation: Style Management and Performance Evaluation* in 1992. In recent years, a style investment strategy has become one of the mainstream models of portfolio management.

At present, generally accepted style analysis methods mainly include income-based style analysis and Combination-Based style analysis. The former is proposed by Sharp. He believes that fund managers' investment style in the past period can be determined by comparing the relationship between fund returns and selected style index returns. The latter mainly divides the investment style of the fund according to the characteristics of the stocks held by the fund. Kahn (1996) found that for small sample funds, portfolio analysis is more relevant to risk prediction than income-based analysis. Kaplan (2003) found that the results of the two style analysis methods are similar for large-scale value portfolio, while for small and medium-sized portfolio and growing portfolio, there are significant differences between the two methods.

Lee and Andrei et al. (1991) used the theory of style investment to explain why funds listed on the same stock market hold totally different stocks, but rise and fall together. Froot et al. (1999) also used the concept of style investment to explain why the same stocks listed on different exchanges perform differently. Sorensen and Lazzara (1995), Anderrson (1997) and Fochtman (1995) have also studied the relationship between a certain style and specific factors (such as macroeconomic factors, price trends, etc.).

In our country, more and more people pay attention to the investment style of the fund and have done some research. Li Ying (2002) made a comprehensive study on the theory, application and investment strategy of style investment. Xiong Shengjun and Yang Chaojun (2003) expounded the theoretical basis of investment style index and introduced the main investment style index and market application. Zeng Xiaojie et al. (2004) found that the investment style of Chinese funds tends to be the same, and the claimed investment style cannot represent its actual style, and analyzed the reasons for this phenomenon. Zhao Hongyu (2005) used the method of portfolio-based style analysis to empirically test the investment style of 30 stock funds. It was found that the actual investment style of many funds violated its declared style. Shi Dayang and Yang Chaojun (2005) used a cluster analysis method to find that the stock market in China has a style based on industry category, and the style of micro-attribute division is not obvious.

## 5. Introducing Portfolio Theory of Transaction Cost and Liquidity

In academic literature, liquidity is generally measured by bid-ask spreads or transaction costs associated with securities trading [15-17]. In practical literature, liquidity is often regarded as the ability of traders to extricate themselves from a position quickly. At this time, poor liquidity is not related to transaction costs, but rather to the number of transactions executed [18].

Documents [19, 20] and so on using the former meaning of liquidity. Davis and Norman use classical stochastic control method to discuss the optimal lifetime consumption problem with proportional transaction cost. The results imply that the rule of thumb is reasonable to "keep the risk part in the zone [a, b], trade only when the boundary is reached, and then make the minimum transaction to avoid leaving the zone", and find that the large transaction (i.e., back to Merton line transaction) is not optimal. Shreve and Sonner [21] generalized Davis and Norman problems by using the technique of viscosity solution. Akian, Sequier and Sulem [22] discuss the finite-term multi-dimensional portfolio problem with the same transaction cost structure and prove that the numerical method is appropriate to solve the multi-asset problem by using the technique of viscosity solution. As an application, they use the finite difference method to solve a three-asset final wealth maximization problem. However, the main problem of the above method is that the strategy still consists of infinitesimal transactions. When there are fixed costs (no matter how much they are), if the holding process reaches the boundaries of the non-trading area, the strategy will lead to huge transaction costs. Eastham and Hastings [23] use impulse control method to solve the portfolio optimization problem when the transaction cost includes the fixed cost part (i.e. the lower limit of the positive transaction cost), and consider the possible constraints on the holding process. However, in their main theorems, there is a need for regularity assumptions about value functions, and their optimization formulas are too restrictive. Korn [24] revised their methods from different aspects. Korn gave a new solution by means of optimal stop, and gave a general method for solving impulse control by solving the corresponding quasi-variable inequality method. For the fixed and proportional transaction cost portfolio problem, it is generally difficult to obtain an analytical solution. For this reason, Korn gives an approximation method, asymptotic analysis method, which is very small but non-zero in transaction cost.

Longstaff [25] focused on the latter meaning of liquidity, focusing on the inherent impact of poor liquidity on trading strategies and securities value. Longstaff develops a continuous-time local equilibrium model under the condition that trading strategies limit investors, and studies the effects of poor liquidity of introducing thin transactions on portfolio decision-making and securities valuation. Since only one valid amount of securities is traded in each issue, investors act as if they are facing borrowing and short selling constraints, even if they are not imposed. However, despite this cautious behavior, investors may still hold positions that are riskier than those that lack liquidity constraints and are optimal. Given the optimal strategy, Longstaff solves the derivative wealth utility of investors and gives the shadow price of illiquidity by comparing the limited and unrestricted wealth utility and solving the asset price discount which compensates for the liquidity limitation of investors. He also pointed out that even without inherent borrowing constraints, the discounts caused by poor liquidity are considerable, and numerical results show that the impact of poor liquidity valuation is in the same order of magnitude as those observed by experience. These results show that the discount observed by bad liquidity experience can be explained in the rational model of investor behavior, i.e. in the rational model, a larger discount is maintained. Of course, this is only the result of local equilibrium and can be regarded as advisory rather than qualitative.

## 6. Continuous Time Portfolio Theory for Long-term Investment

For a long time, Markowitz's mean-variance theory has played an important role in guiding people's short-term investment. But this theory assumes that investors only care about the risks and benefits of a period, which also leads to a series of unreasonable conclusions. In fact, the optimal portfolio of long-term and short-term investments is different. Samuelson (Samuelson, 1963, 1969) first described the constraints on long-term and short-term investors to make the same decision; Merton (Merton, 1969, 1971, 1973) also conducted a long-term, in-depth study of this. Their research tells people that investment opportunities change over time, and long-term investors are always concerned about the impact of long-term investment opportunities and hope to arbitrage from them. As a different theory from short-term investment, long-term investment has been paid more and more attention in the past three decades. However, it is difficult to solve the closed solution of the optimal combination in Merton's intertemporal model, which limits its application. Later, Kim and Omberg (1996), Balduzzi and Lynch (1999), Barberis (2000) established empirical models for portfolio selection of long-term investors. For example, Kim and Omberg studied portfolio selection under continuous time. They assume that an investor with limited life has the HARA utility of end-of-life wealth. They find that no approximation is used and the optimal combination weight is linear. Balduzzi and Lynch tested the utility of investors who neglected the long-term nature of their investments and concluded that ignoring the actual transaction cost would increase the utility cost by 0.8 to 16.9. Barberis's study found that even after incorporating many parameters' uncertainties into the model, there are enough earnings expectations to enable long-term investors always to allocate more assets to stocks.

For the asset allocation of long-term investment, using continuous-time mathematics to analyze dynamic portfolio selection can at least be traced back to the research work of Robert Merton (1969-1973). Duffie (1996), Karatzas (1998) and Shreve (1990) gave the general methods of portfolio selection in continuous time. Chacko and Viceira (1999) discussed the impact of time-varying risk on investment. Cox and Huang (1989), Cox and Leland (1982) and Pliska (1996) put forward the martingale method of cross-period consumption and portfolio selection. By using the SDF (random discount factor) attribute in the complete market, the dynamic problem is transformed into a static one, which makes the result easier to solve.

The research of long-term investment in our country is also gradually attached importance to. Zhang Shuying et al. (1998) discussed the effective progressive boundary of the portfolio and its determination method without considering transaction cost. Fei Weiyin (2001) gives a detailed description of optimal consumer investment, option pricing and dynamic risk measurement. Peng Daheng (2004) deduced that the longer the investment planning period, the greater the proportion of risk assets held by investors. Lu Baoqun and Zhou Wen (2004) considered the optimal portfolio selection problem under the framework of a stochastic process, found that elasticity is the decision variable of portfolio selection, and proposed two stages to solve the optimal portfolio selection problem.

## 7. VaR Portfolio Theory

In addition to using variance and derivative semi-variance and deviation to measure risk, other methods to measure asset risk include stochastic advantage method [26], absolute deviation method [27], lower risk method [28-30] and VaR method. VaR refers to the maximum possible loss under a certain holding period and a certain confidence level. Since VaR was first used by some financial companies to measure the market risk of transactional securities in the 1980s, VaR has been used and concerned by many commercial banks, investment banks, non-financial companies, institutional investors and regulators.

VaR method has changed the insufficiency of portfolio theory in risk measurement to a certain extent, and many scholars have introduced it into the research of portfolio theory. Alexander and Baptista [31-33] examined the combinatorial VaR constraints under VaR-RM environment, i.e. risk managers are limited to the VaR at a predetermined level to maintain wealth during the period. However, these mean-VaR studies do not incorporate VaR-RM into the optimization based on Mean-Variance preference. They only compare the two methods and establish the relationship between mean-variance and mean-VaR effective frontier. When Das and Uppat [34] developed an international portfolio selection model, they constrained the distribution of portfolio returns by imposing an upper bound on the additional peak of the portfolio. They interpreted this constraint as an implicit restriction imposed by investors on the portfolio VaR. Alexander and Baptista [35] characterize the effective frontier of VaR constraints from an analytical point of view, compare it with the unconstrained frontier, and examine the economic implications of using VaR constrained mean-variance model in combination selection. The implication depends not only on the confidence level chosen when calculating the VaR value but also on the risk aversion of the actor.

Basak and Shapiro [36] integrate risk management directly into the utility maximization framework in the continuous time complete market environment. Considering the VaR constraints during the period, the effects of VaR-RM on optimal wealth, consumption choice and optimal portfolio policy are studied. Under constant relative risk aversion preference and logarithmic normal state price, the dynamic portfolio selection of VaR risk managers deviates considerably from the choice of portfolio insurers and benchmark actors, and the deviation is most significant in the transition state with the highest uncertainty of loss occurrence. The general equilibrium analysis reveals that the emergence of VaR risk managers enlarges the volatility of the stock market when the market declines, while weakens the volatility when the market rises. In particular, in most adverse situations, VaR risk managers incur greater losses than non-risk managers. For this reason, Basak and Shapiro proposed the alternative model of risk management, LEL-RM (Minimizing the Risk Management of Expected Loss), where the Expected Loss (not the Probability of Loss) is limited. At the same time, they explained how this alternative model could correct VaR-RM's defect that it focused on loss probability rather than size, and pointed out that in many practical cases, the expected loss of VaR-RM may be 2-10 times larger than that of LEL-RM. Luciano [37] also concentrates on the optimal portfolio selection of utility maximization investors and incorporates VaR management requirements into a class of constraints similar to Basak and Shapiro. When unconstrained optimization solutions are obtained (with or without bid-ask spreads), the bias of constraints is analyzed. This analysis complements Basak and Shapiro because it allows us to examine whether an optimal investor will automatically follow VaR management or the probability that the investor will do so.

## 8. Concluding Remarks

Although the portfolio theory has made great progress in recent years, due to the shortcomings of these advances in disposability and theory itself, most of them remain at the level of theoretical discussion and are seldom applied in practice. For the portfolio theory with liquidity, although there are some solutions to this kind of portfolio problem (such as Atkinson and Wilmott [38], under small but non-zero transaction cost, a perturbation method is given which can be applied to the portfolio problem with about 20-30 assets). However, under the actual combination scale, more theoretical results and algorithms are needed to get the solution of such combinatorial problems. The introduction of heuristic algorithms (such as a genetic algorithm) will be beneficial to the solution of such problems. Compared with Markowitz's combination theory, the combination theory based on VaR has the advantages of improving the efficiency of the model and enhancing the true sense of risk psychology. Its shortcomings lie in the selection of VaR's calculation method, the width and processing method of historical data, etc. The bootstrap statistical method and BVaR (Be-yond the Value-at-Risk) theory can overcome the shortcomings of combination theory based on VaR to some extent. Behavioral portfolio theory explains investors' investment behavior well in reality, but up to now, no effective model has been established for the decision-making of each psychological account of the same investor. The introduction of the behavioral game of several investors will be the future research direction (in practice, it is often to examine the momentum of the whole market or a certain investment group or reverse investment strategy, herding behavior, etc.). In the non-utility-based portfolio theory, the calculation of pan-portfolio theory is tedious, and many definitions have no obvious practical significance. Although the index tracking method has inevitably led to transaction costs and tracking errors due to the change of cash flow, it is still widely used in funds with negative management. Value maintenance portfolio strategy depends on the rationality of value maintenance principle. Like pan-portfolio theory, it is mainly a theoretical discussion, which is far from the actual operation. In view of the fact that the vast majority of funds cannot beat the market confirmed by domestic and foreign research institutes, the index tracking method will be of great use. How to select or construct the index will become the focus of future research.

**References**


[1] Merton R C. Lifetime portfolio selection under uncertainty: the continuous-time case[J]. Review of Economics and Statistics,1969,51:247-257.

[2] Merton R C. Optimum consumption and portfolio rules in a continuous-time model[J]. Journal of Economic Theory,1971,3:373-413.

[3] Shefrin H, Statman M. Behavioral portfolio theory[J]. Journal of Financial and Quantitative Analysis, 2000,35(2):127-151.

[4] Lopes L. Between hope and fear. the psychology of risk[J]. Advances in Experimental Social Psychology,1987,20:255-295.

[5] Kahneman D, Tversky A. Prospect theory: an anal-ysis of decision under risk[J]. Econometrica, 1979,47:263-291.

[6] Tversky A, Kahneman D. Rational choice and the framing of decisions[J]. Journal of Business, 1986,59: S251-S278.

[7] Kroll Y, Levy H, Rapoport A. Experimental tests of the separation theorem and the capital asset pricing model[J]. American Economic Review,1988,78:500-518.

[8] Jorion P. Mean-variance analysis of currency overlays[J]. Financial Analysis Journal,1994,50:48-56.

[9] Thaler R, Shefrin H. The economic theory of self-control[J]. Journal of Political Economy, 1981, 89:392-406.

[10] Cover T. Universal portfolio[J]. Mathematical Finance,1991,1(1):1-29.

[11] Jamshidian F. Asymptotically optimal portfolios[J]. Mathematical Finance,1992,2(2):131-150.

[12] Buckley I RC, Korn R. Optimal cash management for equity index tracking in the presence of fixed and proportional transaction costs [R]. Berichte zur Stochastik und verwandten Gebieten 97-3, Johannes Gutenberg-Universitat Maninz,1997.

[13] Hellwig K. Intertemporal choice reexamined [R]. Discussion Papers in Economics, University of Ulm,1993.

[14] Korn R. Value preserving portfolio strategies in continuous-time models [J]. Mathematical Methods of Operations Research,1997,45(1):1-43.

[15] Amihud Y, Mendelson H. Asset pricing and the BidAsk spread[J]. Journal of Financial Economics, 1986,17:223-249.

[16] Stoll H. Inferring the components of the Bid-Ask spread: theory and empirical tests[J]. Journal of Finance,1989,44:115-134.

[17] Grossman S, Laroque G. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods[J]. Econometria,1999,58:25-52.

[18] Longstaff F A. Optimal portfolio choice and the valuation of illiquid securities[J]. Review of Financial Studies,2001,14(2):407-431.

[19] Davis M H A, Norman A R. Portfolio selection with transaction costs[J]. Mathematics of Operations Research,1990,15(4):676-713.

[20] Morton A J, Pliska S R. Optimal portfolio management with fixed transaction costs[J]. Mathematical Finance, 1995,5(4):337-356.

[21] Shreve S E, Soner H M. Optimal investment and consumption with transaction costs[J]. Annals of Applied Probability,1994, 4(3):609-692.

[22] Akian M, Sequier P, and Sulem A. A finite horizon multidimensional portfolio selection problem with singular Transaction costs[C]. Proceedings of the34th Conference on Decision and Control, 1995, 2193-2198.

[23] Eastham J E, Hasting K J. Optimal impulse control of portfolios[J]. Mathematics of Operations Research,1988,13(4):588-605.

[24] Korn R. Portfolio optimization with strictly positive transaction costs[R]. Berichte zur Stochastik 94-1,Johannes Gutenberg-Universitat Mainz,1994.

[25] Korn R. Optimal portfolios: stochastic models for optimal investment and risk management in continuous time[M]. Singapore: World Scientific Publishing Co.Pte.Ltd.,1997.

[26] Kamara A. Liquidity, taxes, and short-term treasury yields [J]. Journal of Financial and Quantitative Analysis,1994,29:403-417.

[27] Hadar J, Russell W. Rules for ordering uncertain prospects[J]. American Economic Review, 1969, 59 (1):25-34.

[28] Konno H, Yamakazi H. Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market[J]. Management Science,1991,37 (4):519-531.

[29] Roy A D. Safety-first and the holding of assets[J]. Econometrica,1952,20:431-449.

[30] Bawa S Vijay. Optimal rules for ordering uncertain prospects[J]. Journal of Financial Economics, 1975, (2):95-121.

[31] Harlow W V. Asset allocation in a downside risk framework[J]. Financial Analysis Journal, 1991, 47 (5):28-40.

[32] Alexander G, Baptista A. Value at risk and mean variance analysis[R]. Working Paper, University of Minnesota,1999.

[33] Embrechts P, McNeil A, Straumann D. Correlation and Dependency in risk management: properties and pitfalls [R]. Working Paper, Zurich University,1999.

[34] Kast R, Luciano E, Peccati L. Value-at-risk as a decision criterion[R]. Working Paper, University of Turin,1999.

[35] Das S R, Uppal R. The effect of systemic risk on international portfolio choice[R]. Working Paper, Harvard University,1999.

[36] Alexander G, Bapitsta A. A VaR-constrained mean variance model: implications for portfolio selection and the Basle capital accord[R]. Working Paper, University of Minnesota,2001.

[37] Basak S, Shapiro A. Value-at-risk-based risk management: optimal policies and asset prices[J]. Review of Financial Studies,2001,14(2):371-405.

[38] Atkinson C, Wilmott P. Portfolio management with transaction costs: an asymptotic analysis of the Morton and pliska model[J]. Mathematical Finance,1995,5(4):357-367.